VECTOR SPACES

PART-I

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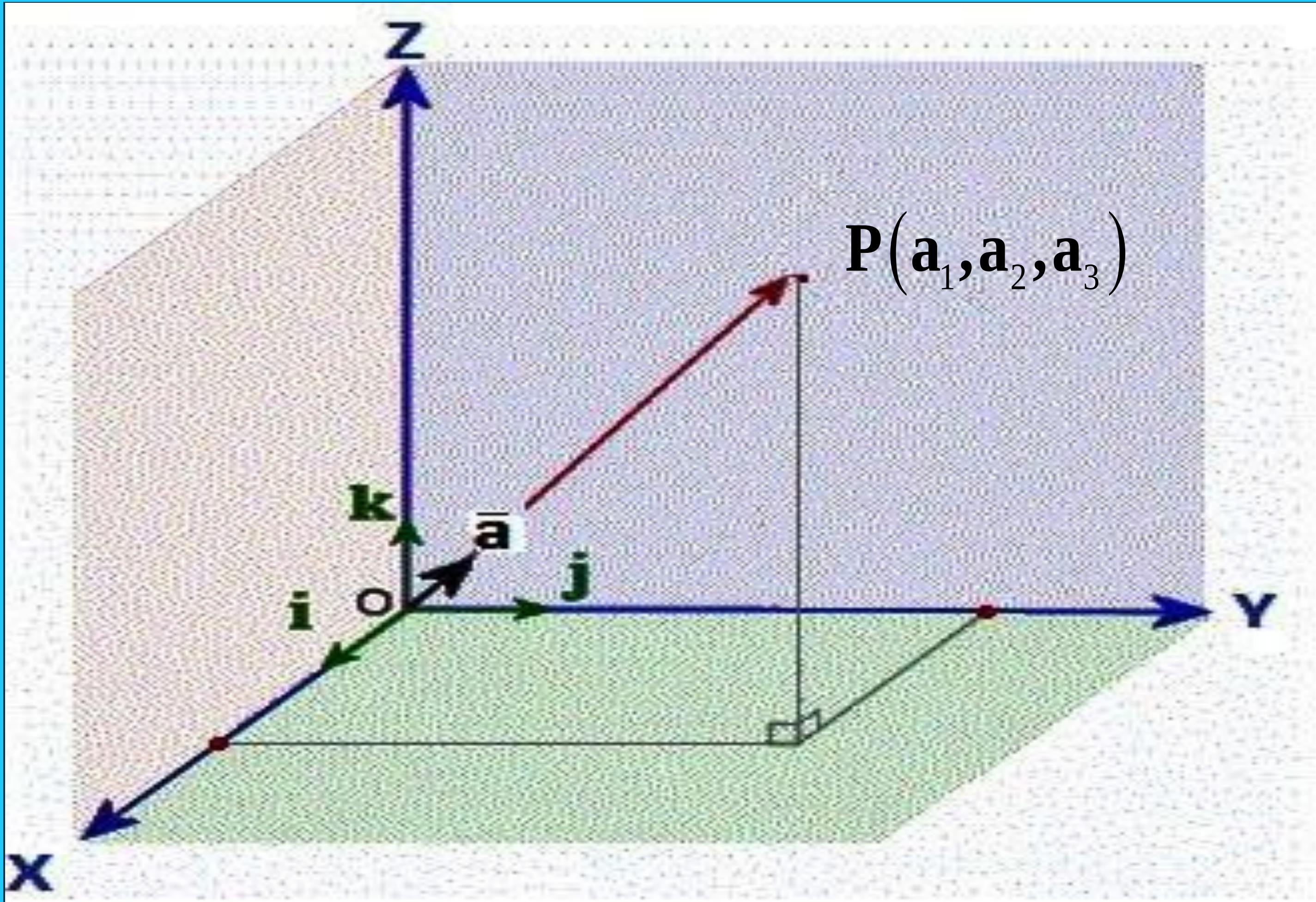
INTRODUCTION

The purpose is to develop a geometric structure, called Vector space or Linear space on which most of the study of mathematical problems and applications is based.

Generally, the idea of vector is with the concept that it has magnitude and direction. Geometrically it is denoted by a line segment.

In science, physical quantities such as force, velocity, displacement and acceleration are considered with magnitude and direction. The term vector is used to identify these entities.

3-D SPACE



$$P = (a_1, a_2, a_3) \Leftrightarrow \overrightarrow{OP} = \bar{a} = (a_1, a_2, a_3)$$

The above diagram reveals that the set of all points in 3-D space has a one-one correspondence with the set of all vectors starting from origin.

Thus each vector is representable as an ordered triad $\left(a_1, a_2, a_3\right)$ of real numbers. We write it as

$$\bar{a} = \overrightarrow{OP} = \left(a_1, a_2, a_3\right)$$

ADDITION OF VECTORS

Let $\mathbf{R}^n$ be an n-dimensional space & $\alpha,\ \beta \in \mathbf{R}^n$,

where $\alpha = \left(\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_\mathbf{n}\right)$ & $\beta = \left(\mathbf{b}_1, \mathbf{b}_2, \cdots, \mathbf{b}_\mathbf{n}\right)$

$$\Rightarrow \alpha + \beta = \left(\mathbf{a}_1 + \mathbf{b}_1,\ \mathbf{a}_2 + \mathbf{b}_2, \cdots\cdots, \mathbf{a}_\mathbf{n} + \mathbf{b}_\mathbf{n}\right)$$

Ex : Take $\alpha = (1, 5, 7)$ and $\beta = (2, 3, 9)$ in $\mathbf{R}^3$

$\Rightarrow \alpha + \beta = (1+2, 5+3, 7+9) = (3, 8, 16)$

SCALAR MULTIPLICATION OF A VECTOR

Let $\mathbf{R}^{\mathbf{n}}$ be an n-dimensional space & $\alpha \in \mathbf{R}^{\mathbf{n}}$,

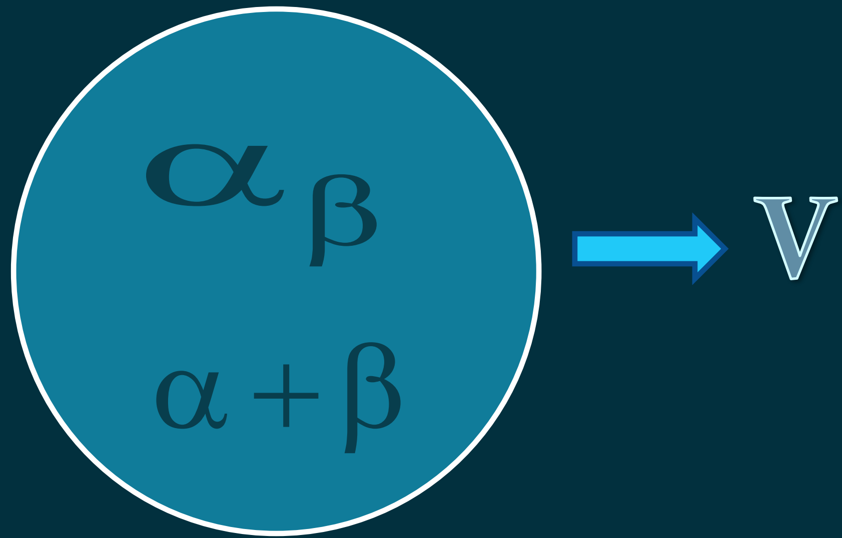
where $\alpha = \left(\mathbf{a}_1, \mathbf{a}_2, \cdots\cdots, \mathbf{a}_{\mathbf{n}}\right)$ & k be a scalar.

$$\Rightarrow \mathbf{k}\alpha = \left(\mathbf{ka}_1, \mathbf{ka}_2, \cdots\cdots, \mathbf{ka}_{\mathbf{n}}\right)$$

Ex : Take $\alpha = (1, -5, 7)$ in $\mathbf{R}^{\mathbf{3}}$ and $\mathbf{k} = 7$

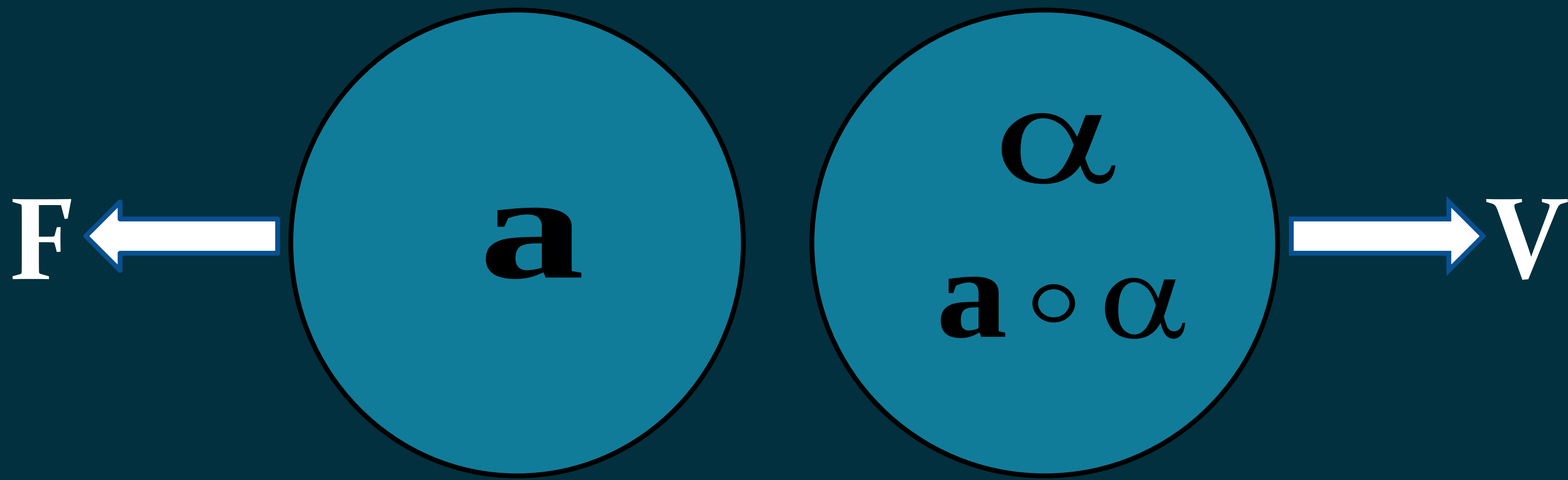
$$\Rightarrow 7\alpha = \left(7(1), 7(-5), 7(7)\right) = (7, -35, 49)$$

Internal Composition



Let V be a set. If $\alpha + \beta \in \mathbf{V} \;\forall\; \alpha, \beta \in \mathbf{V}$ then the operation '+' is said to be an <u>internel composition</u> in V.

External Composition



Let F & V be two sets. If $a \circ \alpha \in V \ \ \forall \ a \in F, \alpha \in V$ then the operation '$\circ$' is said to be an Externel composition in V.

DEFINITION OF FIELD

Let F be a ring. F is said to be a field if

1) F has a unity element

2) Every non-zero element of F having multiplicative inverse

3) F is commutative $\left(\text{OR}\right)$

A commutative division ring is called a field.

Examples : The rings $\left(\mathbf{Q},+,\cdot\right),\left(\mathbf{R},+,\cdot\right)$ and $\left(\mathbf{C},+,\cdot\right)$ are fields.

VECTOR SPACE

Let V be a non-empty set and F be a field.

V is said to be a vector space over F if

1) $(V,+)$ is an abelian group

2) <u>Scalar multiplication:</u> $a \in F, \alpha \in V \Rightarrow a\alpha \in V$

3) <u>Distributive laws</u>

i) $a(\alpha+\beta) = a\alpha + a\beta$

ii) $(a+b)\alpha = a\alpha + b\alpha$

iii) $(ab)\alpha = a(b\alpha)$ for $a, b \in F$ & $\alpha, \beta \in V$

iv) $1\alpha = \alpha \in V$ for $1 \in F, \alpha \in V$

Real Vector space

A vector space $V(F)$ is said to be a real vector space if the field $\underline{F = R}$,

Complex Vector space

A vector space $V(F)$ is said to be a complex vector space if the field $\underline{F = C}$,

ZERO VECTOR SPACE OR NULL SPACE

A vector space $V(F)$ having only one zero vector $\bar{0}$ is called the null space. i.e., $V = \{\bar{0}\}$

EXAMPLES FOR VECTOR SPACE

Ex-I. Take $V = C$ and $F = C$

$$\Rightarrow \underline{C(C) \text{ is a Vector space.}}$$

II. Take $V = R$ and $F = R$

$$\Rightarrow \underline{R(R) \text{ is a Vector space.}}$$

III : Take $V = C$ and $F = R$

$$\Rightarrow \underline{C(R) \text{ is a Vector space.}}$$

IV : The set of all convergent sequences over R is a Vector space.

Q: Is $\mathbf{R(C)}$ a vector space ? Explain.

Ans: NO, $\because \mathbf{V(F)} = \mathbf{R(C)} \Rightarrow \mathbf{V} = \mathbf{R}$ **&** $\mathbf{F} = \mathbf{C}$

Take $\mathbf{a} = 2 + 3\mathbf{i} \in \mathbf{F}$ **and** $\alpha = 5 \in \mathbf{V}$

$$\Rightarrow \mathbf{a}\alpha = 5(2 + 3\mathbf{i}) = 10 + 15\mathbf{i} \notin \mathbf{V}$$

$\therefore \mathbf{R(C)}$ **is a not a vector space.**

$$\left[\begin{array}{l}\textbf{By using scalar multiplication,}\\ \mathbf{a} \in \mathbf{F} \textbf{ and } \alpha \in \mathbf{V} \Rightarrow \mathbf{a}\alpha \in \mathbf{V}\end{array}\right]$$

Note : If V is a vector space over the field F then we write V(F) is a vector space.

STANDARD EXAMPLES

IV : Let V be the set of all ordered n-tuples of <u>any field F</u> for a fixed $+$ve integer n.

$$\therefore \mathbf{V} = \mathbf{V_n}(\mathbf{F}) = \mathbf{F^n} = \{(\mathbf{a_1}, \mathbf{a_2}, \cdots, \mathbf{a_n}) / \mathbf{a_i} \in \mathbf{F}\}$$

$$\textbf{For } \alpha = (\mathbf{a_1}, \mathbf{a_2}, \cdots\cdots, \mathbf{a_n}), \beta = (\mathbf{b_1}, \mathbf{b_2}, \cdots\cdots, \mathbf{b_n})$$

$$\Rightarrow \alpha + \beta = (\mathbf{a_1} + \mathbf{b_1}, \mathbf{a_2} + \mathbf{b_2}, \cdots\cdots\cdots, \mathbf{a_n} + \mathbf{b_n}) \textbf{ and}$$

$$\mathbf{a}\alpha = \mathbf{a}(\mathbf{a_1}, \mathbf{a_2}, \cdots, \mathbf{a_n}) = (\mathbf{aa_1}, \mathbf{aa_2}, \cdots\cdots, \mathbf{aa_n})$$

$$\Rightarrow \mathbf{V_n}(\mathbf{F}) \textbf{ satisfies all axioms in vector space.}$$

$$\therefore \mathbf{V_n}(\mathbf{F}) = \mathbf{F^n} \textbf{ is a Vector space.}$$

<u>Clearly $\mathbf{C^n}$ and $\mathbf{R^n}$ are Vector spaces</u>

Show that the set $V = R^2 = \{(a_1, a_2) / a_1, a_2 \in R\}$ is a vector space w.r.t addition & scalar multiplication defined on V as follows $(a \in R)$

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1 + 1, a_2 + b_2 + 1) \rightarrow (1)$$

$$a(a_1, a_2) = (aa_1 + a - 1, aa_2 + a - 1) \rightarrow (2)$$

I. $(V, +)$ is an abelian group

1. **Closure: Let $\alpha, \beta \in V$**

$\Rightarrow \alpha = (a_1, a_2), \beta = (b_1, b_2)$ where $a_1, a_2, b_1, b_2 \in R$

$\alpha + \beta = (a_1, a_2) + (b_1, b_2) = (a_1 + b_1 + 1, a_2 + b_2 + 1) \in V$

$\Rightarrow$ '$+$' is a binary operation in V.

2. Associate: $\Rightarrow (\alpha+\beta)+\gamma = \alpha+(\beta+\gamma)$

$(\alpha+\beta)+\gamma \quad \because \alpha, \beta, \gamma \in \mathbf{V}$

$= \left[(\mathbf{a}_1, \mathbf{a}_2)+(\mathbf{b}_1, \mathbf{b}_2)\right]+(\mathbf{c}_1, \mathbf{c}_2)$

$= \left[(\mathbf{a}_1+\mathbf{b}_1+1, \mathbf{a}_2+\mathbf{b}_2+1)\right]+(\mathbf{c}_1, \mathbf{c}_2) \because \textbf{using}\,(1)$

$= (\mathbf{a}_1+\mathbf{b}_1+1+\mathbf{c}_1+1, \mathbf{a}_2+\mathbf{b}_2+1+\mathbf{c}_2+1)$

$= (\mathbf{a}_1, \mathbf{a}_2)+\left[(\mathbf{b}_1+\mathbf{c}_1+1, \mathbf{b}_2+\mathbf{c}_2+1)\right]$

$= (\mathbf{a}_1, \mathbf{a}_2)+\left[(\mathbf{b}_1, \mathbf{b}_2)+(\mathbf{c}_1, \mathbf{c}_2)\right]$

$= \alpha+(\beta+\gamma)$

$\therefore$ **The operation '+' is associative in V.**

3. Additive Identity : If $\underline{\alpha+\beta=\beta+\alpha=\alpha}$

then $\beta(=\mathbf{e})$ **is the additive identity in V.**

Take $\alpha+\beta=\alpha$

$$\Rightarrow(\mathbf{a}_1,\mathbf{a}_2)+(\mathbf{b}_1,\mathbf{b}_2)=(\mathbf{a}_1,\mathbf{a}_2)$$

$$\Rightarrow(\mathbf{a}_1+\mathbf{b}_1+1,\mathbf{a}_2+\mathbf{b}_2+1)=(\mathbf{a}_1,\mathbf{a}_2)$$

$$\Rightarrow\mathbf{b}_1=-1,\ \mathbf{b}_2=-1\quad\therefore\beta=(-1,-1)$$

It is easily verified that

$$\alpha+\beta=\beta+\alpha=\alpha$$

$\therefore\beta=\mathbf{e}=(-1,-1)$ **is the additive identity in V**

4. <u>Additive Inverse</u> : If $\alpha+\beta=\beta+\alpha=\mathbf{e}$

then β **is the additive inverse of** α **in V.**

Take $\alpha+\beta=\mathbf{e}$

$$\Rightarrow(\mathbf{a}_1,\mathbf{a}_2)+(\mathbf{b}_1,\mathbf{b}_2)=(-1,-1)\because \textbf{using}(1)$$

$$\Rightarrow(\mathbf{a}_1+\mathbf{b}_1+1,\mathbf{a}_2+\mathbf{b}_2+1)=(-1,-1)$$

$$\Rightarrow\mathbf{a}_1+\mathbf{b}_1+1=-1,\ \mathbf{a}_2+\mathbf{b}_2+1=-1$$

$$\Rightarrow\mathbf{b}_1=-2-\mathbf{a}_1,\ \mathbf{b}_2=-2-\mathbf{a}_2$$

$$\because\beta=(\mathbf{b}_1,\mathbf{b}_2)=(-2-\mathbf{a}_1,\ -2-\mathbf{a}_2)$$

It is easily verified that $\alpha+\beta=\beta+\alpha=\mathbf{e}$

$\therefore(-2-\mathbf{a}_1,\ -2-\mathbf{a}_2)$ **is the additive inverse**

of the element $(\mathbf{a}_1,\mathbf{a}_2)$**.**

5. Abelian: Let $\alpha, \beta \in V \Rightarrow \alpha + \beta = \beta + \alpha$

$$\alpha + \beta = (a_1, a_2) + (b_1, b_2)$$

$$= (a_1 + b_1 + 1, a_2 + b_2 + 1) \because \text{using } (1)$$

$$= (b_1 + a_1 + 1, b_2 + a_2 + 1)$$

$$= \beta + \alpha \quad \because a_1, b_1 \in R \Rightarrow a_1 + b_1 = b_1 + a_1$$

$\therefore (V, +)$ **is an abelian group**

II. Scalar multiplication: Let $a \in R, \alpha \in V$

$$a\alpha = a(a_1, a_2) \qquad \because \text{using } (2)$$

$$= (aa_1 + a - 1, aa_2 + a - 1) \in V$$

$$\left[\because a, a_1 \in R \Rightarrow aa_1 + a - 1, aa_2 + a - 1 \in R\right]$$

$$\therefore a \in R, \alpha \in V \Rightarrow a\alpha \in V$$

III. Distributive Laws:

$$\textbf{Let } \mathbf{a},\mathbf{b} \in \mathbf{R} \ \& \ \alpha,\beta \in \mathbf{V} \Rightarrow \textbf{i) } \mathbf{a}\left(\alpha+\beta\right) = \mathbf{a}\alpha + \mathbf{a}\beta$$

$$\mathbf{LHS} = \mathbf{a}\left(\alpha+\beta\right) = \mathbf{a}\left[\left(\mathbf{a}_1,\mathbf{a}_2\right)+\left(\mathbf{b}_1,\mathbf{b}_2\right)\right]$$

$$= \mathbf{a}\left(\mathbf{a}_1+\mathbf{b}_1+1,\ \mathbf{a}_2+\mathbf{b}_2+1\right) \because \textbf{using}\left(1\right)\&\left(2\right)$$

$$= \left(\underline{\mathbf{aa}_1+\mathbf{ab}_1+\mathbf{a}}+\mathbf{a}-1,\ \underline{\mathbf{aa}_2+\mathbf{ab}_2+\mathbf{a}}+\mathbf{a}-1\right)$$

$$= \left(\mathbf{aa}_1+\mathbf{ab}_1+2\mathbf{a}-1,\ \mathbf{aa}_2+\mathbf{ab}_2+2\mathbf{a}-1\right)$$

$$\mathbf{RHS} = \mathbf{a}\alpha + \mathbf{a}\beta$$

$$= \mathbf{a}\left(\mathbf{a}_1, \mathbf{a}_2\right) + \mathbf{a}\left(\mathbf{b}_1, \mathbf{b}_2\right) \qquad \because \textbf{using}\left(2\right)$$

$$= \left(\mathbf{aa}_1 + \mathbf{a} - 1,\ \mathbf{aa}_2 + \mathbf{a} - 1\right) +$$

$$\left(\mathbf{ab}_1 + \mathbf{a} - 1,\ \mathbf{ab}_2 + \mathbf{a} - 1\right) \qquad \because \textbf{using}\left(1\right)$$

$$= \left(\mathbf{aa}_1 + \mathbf{a} - 1 + \mathbf{ab}_1 + \mathbf{a} - 1 + 1,\ \right) +$$

$$\left(\mathbf{aa}_2 + \mathbf{a} - 1 + \mathbf{ab}_2 + \mathbf{a} - 1 + 1\right)$$

$$= \left(\mathbf{aa}_1 + \mathbf{ab}_1 + 2\mathbf{a} - 1\right) + \left(\mathbf{aa}_2 + \mathbf{ab}_2 + 2\mathbf{a} - 1\right)$$

$$= \mathbf{RHS} \qquad \therefore \mathbf{LHS} = \mathbf{RHS}$$

It is easily verified that,

ii) $\left(\mathbf{a}+\mathbf{b}\right)\alpha = \mathbf{a}\alpha + \mathbf{b}\alpha$

iii) $\left(\mathbf{ab}\right)\alpha = \mathbf{a}\left(\mathbf{b}\alpha\right)$

iv) $1\alpha = \alpha$ **for** $\mathbf{1} \in \mathbf{R}$

$\therefore \mathbf{V}\left(\mathbf{R}\right)$ **satisfies all axioms in vector space.**

$\Rightarrow \mathbf{V}\left(\mathbf{R}\right)$ **is a vector space.**

Hence completed

PROBLEM

S.T the set $V = R^2 = \{(a, b) / a, b \in R\}$ is not a vector space over R w.r.t the operations of addition & scalar multiplication defined as

$$(a_1, b_1) + (a_2, b_2) = (a_1 + a_2, 0)$$

$$c(a_1, b_1) = (ca_1, 0)$$

where $(a_1, b_1), (a_2, b_2) \in V$ and $c \in R$

Sol: Let $\alpha, \beta \in \mathbf{V} \Rightarrow \alpha = (\mathbf{a}_1, \mathbf{b}_1), \beta = (\mathbf{a}_2, \mathbf{b}_2)$

Addition & scalar multiplication is defined as

$$(\mathbf{a}_1, \mathbf{b}_1) + (\mathbf{a}_2, \mathbf{b}_2) = (\mathbf{a}_1 + \mathbf{a}_2, 0) \qquad \rightarrow (1)$$

$$\mathbf{c}(\mathbf{a}_1, \mathbf{b}_1) = (\mathbf{ca}_1, 0), \ \mathbf{c} \in \mathbf{R} \qquad \rightarrow (2)$$

By using the definition of a Vector space,

$1\alpha = \alpha$ **for 1** $\in$ **F and** $\alpha \in \mathbf{V} \quad (\because \mathbf{F} = \mathbf{R})$

But $1\alpha = 1(\mathbf{a}_1, \mathbf{b}_1) = (1\mathbf{a}_1, 0) = (\mathbf{a}_1, 0) \neq \alpha$

$\therefore$ **V is not a vector space over a field R.**

PROBLEM

S.T the set $V = R^2 = \{(a, b) / a, b \in R\}$ is not a vector space over R w.r.t the operations of addition & scalar multiplication defined as

$$(a_1, b_1) + (a_2, b_2) = (a_1 + a_2, b_1 + b_2)$$

$$c(a_1, b_1) = (|c|a_1, |c|b_1)$$

where $(a_1, b_1), (a_2, b_2) \in V$ and $c \in R$.

<u>Solution</u>: Let $\alpha, \beta \in \mathbf{V} \Rightarrow \alpha = (\mathbf{a}_1, \mathbf{b}_1), \beta = (\mathbf{a}_2, \mathbf{b}_2)$

Addition & scalar multiplication is defined as

$$(\mathbf{a}_1, \mathbf{b}_1) + (\mathbf{a}_2, \mathbf{b}_2) = (\mathbf{a}_1 + \mathbf{a}_2, \mathbf{b}_1 + \mathbf{b}_2) \qquad \to (1)$$

$$\mathbf{c}(\mathbf{a}_1, \mathbf{b}_1) = (|\mathbf{c}|\mathbf{a}_1, |\mathbf{c}|\mathbf{b}_1), \ \mathbf{c} \in \mathbf{R} \qquad \to (2)$$

By using Distributive law

$(\mathbf{a}+\mathbf{b})\alpha = \mathbf{a}\alpha + \mathbf{b}\alpha$ **for** $\mathbf{a},\mathbf{b} \in \mathbf{F}, \alpha \in \mathbf{V} \quad (\because \mathbf{F} = \mathbf{R})$

$\mathbf{LHS} = (\mathbf{a}+\mathbf{b})\alpha = (\mathbf{a}+\mathbf{b})(\mathbf{a}_1, \mathbf{b}_1) = (|\mathbf{a}+\mathbf{b}|\mathbf{a}_1, \ |\mathbf{a}+\mathbf{b}|\mathbf{b}_1)$

$\mathbf{RHS} = \mathbf{a}\alpha + \mathbf{b}\alpha = \mathbf{a}(\mathbf{a}_1, \mathbf{b}_1) + \mathbf{b}(\mathbf{a}_1, \mathbf{b}_1)$

$= (|\mathbf{a}|\mathbf{a}_1, |\mathbf{a}|\mathbf{b}_1) + (|\mathbf{b}|\mathbf{a}_1, |\mathbf{b}|\mathbf{b}_1) = \left[(|\mathbf{a}|+|\mathbf{b}|)\mathbf{a}_1, \ (|\mathbf{a}|+|\mathbf{b}|)\mathbf{b}_1\right]$

here $\mathbf{LHS} \neq \mathbf{RHS} \qquad \because |\mathbf{a}+\mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$

$\therefore$ **V is not a vector space.**

TRY TO SOLVE THESE PROBLEMS

Show that the set $V = R^2 = \{(a, b) / a, b \in R\}$

is not a vector space over R w.r.t the operations

of addition & scalar multiplication defined as

1. $(a, b) + (c, d) = (0, b + d),\ k(a, b) = (ka, kb)$
2. $(a, b) + (c, d) = (a + c, b + d),\ k(a, b) = (0, kb)$
3. $(a, b) + (c, d) = (a + c, b + d),\ k(a, b) = (ka, 0)$

where $(a, b), (c, d) \in V$ and $k \in R$.

GENERAL PROPERTIES

Let $V(F)$ be a vector space and a, b $\in$ F, $\alpha, \beta \in V$ & $\bar{0} \in V$, $0 \in F$. Then prove that

i) $\mathbf{a} \cdot \bar{0} = \bar{0}$ **ii)** $0 \cdot \alpha = \bar{0}$ **iii)** $\mathbf{a}(\alpha - \beta) = \mathbf{a}\alpha - \mathbf{a}\beta$

iv) $(\mathbf{a} - \mathbf{b})\alpha = \mathbf{a}\alpha - \mathbf{b}\alpha$ **v)** $\mathbf{a}\alpha = \bar{0} \Rightarrow \mathbf{a} = 0$ **or** $\alpha = \bar{0}$

vi) $\mathbf{a}\alpha = \mathbf{b}\alpha \Rightarrow \mathbf{a} = \mathbf{b}$ **if** $\alpha \neq \bar{0}$

I. Let $V(F)$ be a vector space and $0 \in F, \bar{0} \in V$

$\because \bar{0} \in V \Rightarrow \bar{0} + \bar{0} = \bar{0} \quad \Rightarrow \mathbf{a} \cdot (\bar{0} + \bar{0}) = \mathbf{a} \cdot \bar{0}$

$\Rightarrow \mathbf{a} \cdot \bar{0} + \mathbf{a} \cdot \bar{0} = \mathbf{a} \cdot \bar{0} + \bar{0} \; [\mathbf{a} \in F, \alpha \in V \Rightarrow \mathbf{a}\alpha \in V]$

$\Rightarrow \mathbf{a} \cdot \bar{0} = \bar{0}$ [using LCL]

II. Let $\mathbf{V}(\mathbf{F})$ be a vector space and $0 \in \mathbf{F}, \bar{0} \in \mathbf{V}$

$$\because 0 \in \mathbf{F} \Rightarrow 0+0=0 \Rightarrow (0+0)\cdot\alpha = 0\cdot\alpha$$

$$\Rightarrow 0\cdot\alpha + 0\cdot\alpha = 0\cdot\alpha + \bar{0} \Rightarrow 0\cdot\alpha = \bar{0} \ (\textbf{using LCL})$$

NOTE $\Rightarrow$

$$\alpha \in \mathbf{V} \Rightarrow \alpha = \alpha + \bar{0} = \bar{0} + \alpha$$

$$\mathbf{a} \in \mathbf{F} \Rightarrow \mathbf{a} = \mathbf{a} + 0 = 0 + \mathbf{a}$$

III. <u>We prove that $\mathbf{a}(\alpha-\beta) = \mathbf{a}\alpha - \mathbf{a}\beta$</u>

$$\textbf{LHS} = \mathbf{a}(\alpha-\beta) = \mathbf{a}\left[\alpha + (-\beta)\right] = \mathbf{a}\alpha + \mathbf{a}(-\beta)$$

$$= \mathbf{a}\alpha + \left[-(\mathbf{a}\beta)\right] = \mathbf{a}\alpha - \mathbf{a}\beta = \textbf{RHS}$$

IV. <u>We prove that $(a-b)\alpha = a\alpha - b\alpha$</u>

$$LHS = (a-b)\alpha = [a+(-b)]\alpha = a\alpha + (-b)\alpha$$

$$= a\alpha + [-(b\alpha)] = a\alpha - b\alpha = RHS$$

V. suppose $a \neq 0 \Rightarrow a^{-1}$ exists & $aa^{-1} = a^{-1}a = 1$

Take $a\alpha = \bar{0} \Rightarrow a^{-1}(a\alpha) = a^{-1}\bar{0} \quad \Rightarrow (a^{-1}a)\alpha = \bar{0}$

$\Rightarrow \alpha = \bar{0} \qquad \therefore a \neq 0$ and $a\alpha = \bar{0} \Rightarrow \alpha = \bar{0}$

VI. <u>We p.t $a\alpha = b\alpha \Rightarrow a = b$ if $\alpha \neq \bar{0}$</u>

For $a\alpha = b\alpha \quad \Rightarrow (a-b)\alpha = \bar{0}$

$\Rightarrow a - b = 0 \quad \Rightarrow a = b \quad (\because \alpha \neq \bar{0})$

Hence proved

ADDITIONAL INFORMATION

https://www.math.tamu.edu/~dallen/m640_03c/lectures/chapter1.pdf

http://www.maths.usyd.edu.au/u/bobh/UoS/MATH2902/vswhole.pdf

https://www.youtube.com/watch?v=ozwodzD5bJM

https://www.youtube.com/watch?v=_6oRqxY6O5w

https://www.youtube.com/watch?v=gcRbNKMpnXc

https://www.youtube.com/watch?v=vNKIs0UStH0

CONCLUSION

In this presentation, you will be expected to learn several things about vector spaces perhaps even more importantly, you will be expected to acquire the ability to think clearly and express yourself clearly, for this is what mathematics is really all about.

Thank you all, for listening to me so far.

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